

# Performance of Space-Time Block Coded MIMO Systems with Antenna Selection

A.S. Hiwale and A. A. Ghatol

**Abstract-** Multi-antenna systems can be used for increased capacity or for increasing diversity order, but the cost paid for deriving these benefits is increased hardware complexity due to multiple antennas and the number of RF (radio frequency) chains. A well-known promising technique to reduce the hardware complexity is to use antenna selection for selecting a subset of available antennas that have stronger links compared to others. This paper, gives an overview of principles of antenna selection in multiple-input multiple-output (MIMO) systems. The impact of antenna selection on the data rate/channel capacity and the performance of MIMO system employing space-time coding have been discussed. Orthogonal space-time block codes (OSTBC) are considered for the capacity and error probability analysis of MIMO systems with receive antenna selection. The channel model considered is the Rayleigh fading channel as it represents the most practical non-line of sight (N-LOS) channel. It is seen that the diversity order is maintained with antenna selection with little loss in channel capacity.

**Indexing Terms:** Multiple-Input Multiple-Output (MIMO), Bit Error Rate, Channel Capacity, Rayleigh fading channels, Antenna selection

## I. INTRODUCTION

WIRELESS communications has made a tremendous impact on the lifestyle of a human being. It is very difficult to survive without wireless in some form or the other. As compared to fixed wireless systems, today's wireless networks provide high-speed mobility (mobile users in fast vehicles) for voice as well as data traffic. Continuous exponential growths of Internet, Cellular Mobile and Multimedia Services in the near past are the driving forces for the increased demand of data rates in Wireless Communication Networks. The availability of limited spectrum on one side and ever increasing demand for increased data rates and quality of service (QoS) on the other side, follows the quest for spectrally efficient signaling techniques. Future wireless systems are expected to support variety of new services, which will demand extensively high data rates. Hence high data rate wireless communications, approaching 1 Gbps, is of great interest and the major focus of wireless research community is directed towards meeting gigabit transmission over wireless systems. MIMO technology is an attractive solution that offers substantial leverages in achieving gigabit wireless links a reality.

The key issues to be dealt in the design of wireless communication system are fading and interference. Conventional wireless system design treats fading and interference as nuisances as far as improving the reliability of air interface is concerned. Recently the designers' focus has shifted towards increasing spectral efficiency, where fading is viewed as opportunity to be exploited to design spectrally efficient systems. MIMO systems use multiple antennas at both ends of communication link and have shown considerable increase in spectral efficiency, suppress interference and improve the reliability of transmission [1-3]. Because of these features, MIMO systems have received a great attention of wireless research community in the last decade. Simple practical MIMO scheme proposed by [4] is already implemented in 3G and WLAN standards and the use of its extensions are being analyzed for next generation wireless systems.

Despite all these advantages, MIMO systems have not been yet adopted widely in practical wireless systems. One of the reasons is the high implementation cost, because of using separate RF chain for each employed antenna. While antenna elements are cheap (usually a patch or dipole in the form of a metallic rod or piece of copper), RF chains increase the implementation cost significantly. Those wireless systems where MIMO are adopted (3G cellular, 802.11n, 802.16, etc), the number of antennas is limited to a few.

Antenna selection techniques have emerged in recent past as a means to mitigate the hardware complexity problem while retaining the benefits provided by transmit and receive antennas. In antenna selection, the number of available antennas at the transmitter and the receiver are large than the number of RF chains. The idea is, RF chains are connected to the antennas having strong link between transmitter and receiver. Antenna selection can be performed at the receiver only (RAS), at the transmitter only (TAS) or at both the transmitter and the receiver (TR-AS). However, the most commonly used antenna technique is receive antenna selection (RAS), the reason being TAS and TR-AS requires the knowledge of channel feedback which works well only when channel changes very slowly. The analytical results given in this paper considers receive antenna selection due to its superiority and practicality over TAS and TR-AS. A brief look into various antenna selection techniques has been covered and the impact of RAS on channel capacity and performance with OSTBC is presented assuming Rayleigh fading channel.

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## II. SYSTEM MODEL

The generic block diagram of the system considered is shown in Figure 1, consisting of  $N_T$  transmit and  $N_R$  receive antennas whereas  $N_t$  transmit and  $N_r$  receive RF chains. In MIMO system having number of RF chains at the transmitter and receiver equal to the number of antennas at the transmitter and receiver respectively, the incoming data is encoded by the space-time encoder and fed to serial-to-parallel converter to convert the input bit stream into  $N_T$  parallel streams. These  $N_T$  streams are transmitted from  $N_T$  transmit antennas simultaneously [5-6].

The received signal vector is given by

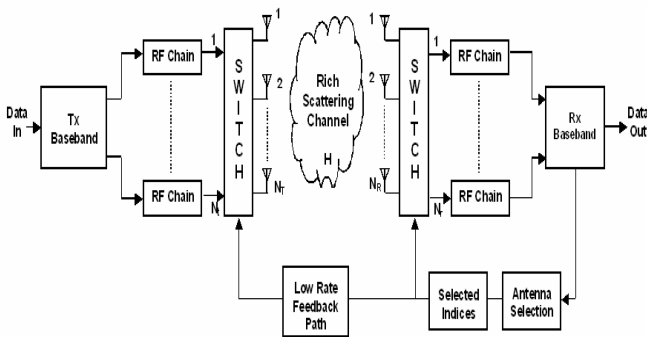
$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (1)$$

where  $\mathbf{H}$  is  $N_R \times N_T$  complex channel matrix representing the uncorrelated channel. The  $ij^{\text{th}}$  entry of matrix  $\mathbf{H}$  denoted by  $h_{ij}$  represents the channel fading coefficient from the  $i^{\text{th}}$  transmit antenna to the  $j^{\text{th}}$  receive antenna,  $\mathbf{x}$  is  $N_T \times 1$  column vector and  $\mathbf{n}$  is  $N_R \times 1$  column vector representing AWGN noise samples. The entries of  $\mathbf{H}$  are modeled as independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and variance 0.5 per dimension and the elements of  $\mathbf{n}$  are modeled as i.i.d. complex Gaussian random variables with zero mean and variance  $N_o/2$  per dimension. Rayleigh distribution is the most representative of Non-Line of Sight (N-LOS) wireless radio propagation and hence the MIMO channel capacity has been investigated for Rayleigh fading channel model. It is assumed that the channel state information (CSI) is known exactly at the receiver and not at the transmitter, and the channel fading coefficients remain constant over the entire frame and changes from one frame to another.

The information-theoretic capacity of such full complex MIMO systems which use all available transmit and receive antennas is given by

$$C_{full} = \left[ \log_2 \det \left( I_{N_R} + \frac{E_s}{N_T N_o} \mathbf{H} \mathbf{H}^H \right) \right] \quad \text{if } N_R < N_T \quad (2)$$

where  $\mathbf{H}^H$  is component wise transpose conjugate of  $\mathbf{H}$ .  $I_{N_R}$  is



$N_R \times N_R$  identity matrix.

Figure 1. Block diagram of MIMO system with antenna selection

## III. SPACE-TIME CODES

MIMO systems provide information about theoretically increased spectral efficiency. Coded modulation schemes for MIMO wireless channels are popularly known as Space-Time codes. Space-Time codes are the practical signal design techniques to realize the information theoretic capacity limits of MIMO system. Space-Time codes perform coding in both spatial and temporal domains to introduce correlation between signals transmitted from different transmit antennas at various time periods. Tarokh *et al* first introduced generalized space-Time codes in 1998. These codes allow achieving diversity gain and coding gain over spatially uncoded systems.

Space-Time Block Codes (STBC) and Space-Time Trellis codes (STTC) are the two types of most commonly used space-time codes. STTCs produce a sequence of vector symbols of length equal to the number of transmit antennas. In addition to diversity gain inherent in space-time codes, STTC provides coding gain like traditional Trellis Coded Modulation (TCM). In contrast to STTC, STBC produce matrix output with the rows and columns representing the transmit antenna index and the time index respectively. STBCs provide diversity gain but do not provide coding gain. In orthogonal STBCs, the vectors representing any pair of columns taken from coding matrix are orthogonal, resulting into simple, linear and optimal decoding at the receiver. Because of simple design and low complexity receivers, OSTBCs are preferred as compared to STTCs. Most of the applications that have adopted MIMO in their standards use OSTBC as an option. OSTBC may be combined with an outer channel code (convolutional code or TCM as an outer code) to provide coding gain in addition to diversity benefit. This approach is called as concatenation scheme, but adding an outer code reduces the spectral efficiency.

## IV. ANTENNA SELECTIONS

Different antenna selection schemes are given in [7–15]. The fundamental antenna selection approaches are based on whether the spatial multiplexing (maximizing the channel capacity) or spatial diversity (maximizing the instantaneous output signal-to-noise ratio at the receiver) is used.

### A. Antenna Selection Based on Channel Capacity

In case of antenna selection at receiver only, it is considered that the number of RF chains at the transmitter is equal to the total number of antennas  $N_T$  and  $N_r \leq N_R$ . The incoming data is encoded by the space-time encoder to obtain  $N_T$ -element transmit signal vector and mapped onto selected  $N_T$  transmit antennas. The transmitted signal vector then travel through MIMO propagation channel represented by modified channel matrix  $\mathbf{H}$  which is having dimension  $N_r \times N_T$ .

With antenna selection, the information theoretic capacity of MIMO is given by

$$C_{sel} = \max_{S(H)} \left[ \log_2 \det \left( I_{N_r} + \frac{E_s}{N_T N_o} \mathbf{H} \mathbf{H}^H \right) \right] \quad (3)$$

where rows and columns of  $H$  corresponds to the selected antennas at the receiver and transmitter respectively and  $S(H)$  is the set of all possible sub-matrices of  $H$ . One of the key consequences of equation (3) is that the average capacity, determining the possible number of data streams, increases linearly with minimum  $(N_T, N_r)$ . It can be seen from the plots shown in section for results that the upper bound on channel capacity,  $C_{sel}$ , is tight for  $N_r < N_T$ , but it is loose for  $N_r > N_T$ .

### B. Antenna Selection Based On Received SNR

Frobenius norm based selection scheme is a simple, yet effective antenna selection scheme to select receive antennas, corresponding to rows of  $H$  having maximum Frobenius norm. The relationship between the SNR ( $\gamma$ ), error probability and Frobenius norm has been very well studied in [7-9] and are as given below

$$\gamma = \frac{E_s}{N_T N_o} \|H\|_F^2 \quad (4)$$

$$\text{and } P_e \leq \exp\left[-\frac{E_s}{N_T N_o} \|H\|_F^2\right] \quad (5)$$

where  $E_s$  is the average signal energy per transmitted symbol,  $N_o$  is the noise power and  $\|H\|_F^2$  is Frobenius norm of channel matrix  $H$ , given by  $\|H\|_F^2 = \sum_{i=1}^{N_T} \sum_{j=1}^{N_R} |h_{ij}|^2$ . It can be seen from

(4) and (5) that maximizing channel Frobenius norm maximizes the SNR and minimizes the instantaneous error probability. Gore *et al.* in [7] has used this logic for developing optimal antenna selection algorithm. In this paper, the norm based selection criteria for antenna selection at receiver only, is considered.

The bit error rate (BER) of MIMO with BPSK signaling and ML decoding at high SNRs can be approximated as

$$P_b(e) \cong \binom{2N_T N_R - 1}{N_T N_R} (4\gamma)^{-N_T N_R} \quad (6)$$

where  $\gamma$  is signal-to-noise ratio and the diversity order is  $N_T N_R$ . When antenna selection is used to select best  $N_r$  antennas out of  $N_R$ , the BER can be approximated as

$$P_b(e) \cong \binom{2N_T N_R - 1}{N_T N_R} \left(4 \frac{N_r}{N_R} \gamma\right)^{-N_T N_R} \quad (7)$$

Comparing Eq. (6) and (7) shows that, diversity order is maintained with antenna selection, as that of full complexity system for any  $N_r < N_R$ . Also it can be shown that SNR due to antenna selection is upper bounded by  $10 \log_{10}(N_R / N_r)$  dB. The results shown by Eq. (6) and (7) hold for all fading channel models with the use of OSTBCs, whereas the results

vary for different fading models with the use of STTCs. This is the major advantage of OSTBC over STTC.

## V. NUMERICAL RESULTS

Figure 2 shows MIMO capacity without and with receive antenna selection for  $N_T = 2$ ,  $N_R = 2$  and  $N_r = 1, 2$ . With the full exploitation of all available antennas, 6 b/s/Hz can be transmitted over the channel at 15 dB. The capacity decreases as the number of selected antenna elements  $N_r$  of  $N_R$  are decreased, reaching 5 b/s/Hz at  $N_r = 1$ . Similarly with  $N_T = 2$ ,  $N_R = 4$  and  $N_r = 1, 2, 3, 4$ . The capacity decreases gradually as the number of selected antenna elements  $N_r$  of  $N_R$  is decreased, but the capacity decreases drastically (the bound is tight) for  $N_r < N_T$ , which can be seen from Figure 4.

In Figure 3, 5 and 6, plots of BER against  $E_b/N_o$  in decibels for the cases 2X2, 2X4 and 4X4 systems selecting  $N_r = 1$  to  $N_R$  antennas. Comparing the result of without antenna selection and with antenna selection, it can be seen that the slope of BER characteristics remains unchanged indicating that the diversity order is maintained with antenna selection as that of full complex system using all available antennas. Figure 6 shows that the error probability is improved in case of 4X4 system compared to that of 2X4 system.

## VI. CONCLUSION

In this paper, the capacity and performance of space-time block coded MIMO systems have been analyzed, with and without antenna selection at the receiver. It is observed that the achieved capacity with antenna selection is close to the capacity of full complexity system for  $N_r \geq N_T$ . The diversity order with antenna selection is maintained as that of the full complexity system. Since the cost of transmit/receive RF chains is a significant factor limiting the use of multiple antennas in wireless link, we believe that antenna subset selection, which is a low-cost low-complexity technique that retains the diversity benefits of multiple antennas, is a promising solution.

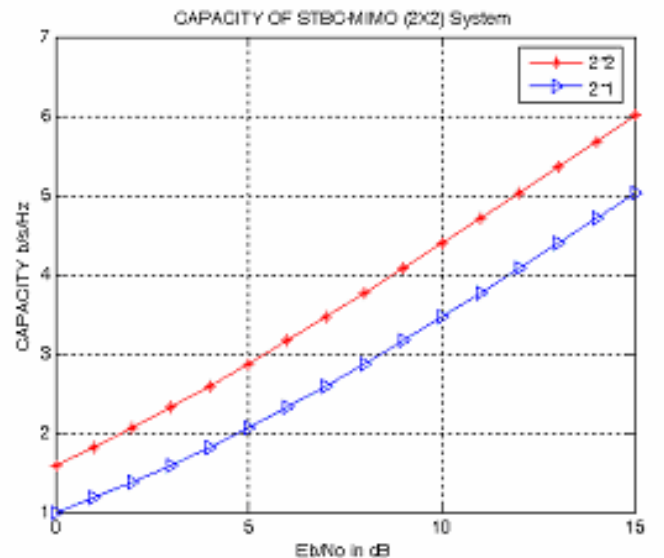


Fig 2. Capacity of 2X2 MIMO system with receive antenna selection

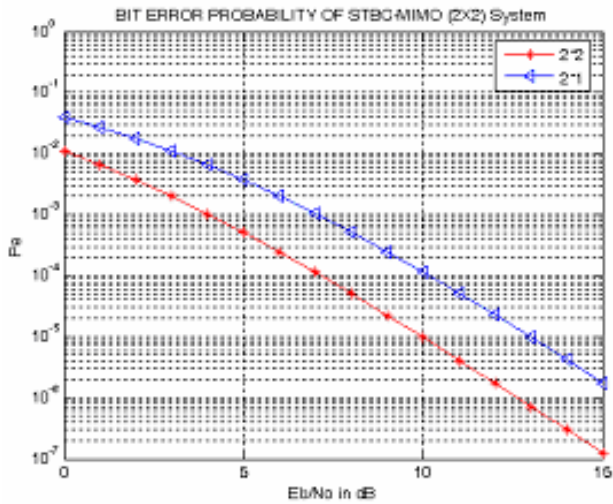


Fig.3 Error probability of 2X2 MIMO system with receive antenna selection

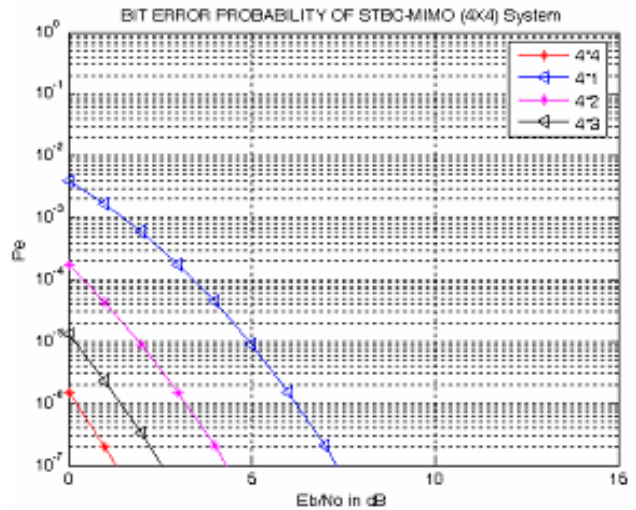


Fig. 6 Error probability of 4X4 MIMO system with receive antenna selection

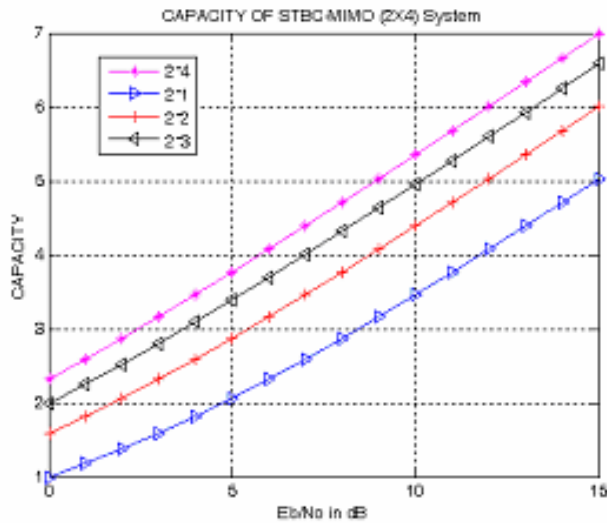


Fig.4 Capacity of 2X4 MIMO system with receive antenna selection

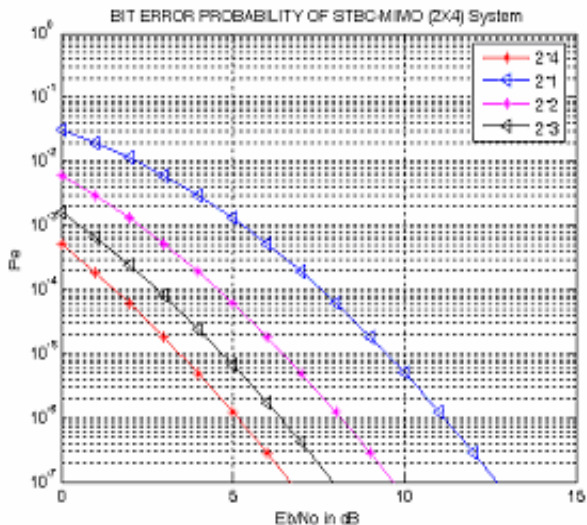


Fig. 5 Error probability of 2X4 MIMO system with receive antenna selection

VII. REFERENCES

- [1] G.J. Foschini and M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas", *Wireless Personal Communications*, vol. 6, pp. 311–335, 1998.
- [2] A. F. Molisch, M. Z. Win and J. H. Winters, "Capacity of MIMO systems with antenna selection", in *Proc. IEEE Int. Conf. Comm.*, June 2001, pp. 570-574.
- [3] D. Gesbert, M. Shafi, D. Shan Shu, P. J. Smith and A. Naguib, "From theory to practice: an overview of MIMO space-time coded wireless systems", *IEEE Journal Select. Areas Commun.*, vol. 21, pp. 281–302, 2003.
- [4] S. M. Alamouti, "A simple transmit diversity technique for wireless communications", *IEEE Journal Select. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.
- [5] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance analysis and code construction," *IEEE Trans. Inform. Theory*, vol. 44, no. 2, pp. 744–765, Mar. 1998.
- [6] V. Tarokh, H. Jafarkhani and A. R. Calderbank, "Space-time block codes from orthogonal designs", *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456–1467, July 1999.
- [7] D. A. Gore, A. Paulraj, "MIMO antenna subset selection with space-time coding", in *IEEE Trans. On Signal Processing.*, vol. 50, No. 10, pp. 2580-2588, October 2002.
- [8] R. Machado and B. F. Uchoa-Filho, "Space-time block coding with hybrid antenna /code selection", in *Proc. IEEE ICC 2004*, pp. 819-822.
- [9] A. Ghrayeb, T. M. Duman, "Performance analysis of MIMO systems with antenna selection over quasi-static fading channels", in *IEEE Trans. Veh. Technology.*, vol. 52, No. 2, pp. 281-288, 2003.
- [10] A. Ghrayeb, "A Survey on Antenna Selection for MIMO Communication Systems", *IEEE Proc. of ICTTA'06, Vol. 2*, pp. 2104-2109.
- [11] X Zeng, A. Ghrayeb, "Performance bounds for space-time block codes with receive antenna selection", *IEEE Trans. Inform. Theory*, vol. 90, no. 9, pp. 2130–2137, September 2004.
- [12] Z. Chen, B. Vucetic and K. L. Lo, "Analysis of transmit antenna selection/Maximal-Ratio combining in Rayleigh fading channels", *IEEE Proc. of ICCT2003*, pp. 1532-1536, 2003.
- [13] Z. Chen, B. Vucetic and J. Yuan, "Asymptotic performance of space-time block codes with imperfect transmit antenna selection", *IEEE Electronics Letters* vol. 41, no. 9, April 2005.
- [14] W. H. Wong, and E. G. Larsson, "Orthogonal space-time block coding with antenna selection and power control", *IEEE Electronics Letters*, vol. 39, no. 4, pp. 379-381, Feb. 2003.
- [15] A. Gorokhov, D. Gore and A. Paulraj, "Receive antenna selection for MIMO flat-fading channels: theory and algorithms", *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2687-2696, October 2003.

## VIII. BIOGRAPHIES



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