Improved performance of Irregular LDPC Codes by attaining regularity for AWGN channel

Kuldeep Yadav and M.C. Srivastava

Abstract--**The study in this paper has been emphasized to investigate the performance of Irregular low-density parity check (LDPC) codes trying to attain regularity by various rearrangements techniques as proposed by Radford Neal[2].The LDPC encoded data is modulated using various modulation techniques viz. PSK and QAM. To improve the performance for bandwidth efficient transmission, the modulated data is pulse shaped using a Square Root Raised Cosine filter and reshaped on the receiver. The decoder designed uses Forward Backward and Downward Upward passes based on Sum product algorithm which provides a better BER performance for large data size. A considerable improvement in results has been attained as shown in results.**

Index Terms--**Parity check matrix, sparse matrices, encoding, modulation, pulse shaping, decoding, irregular LDPC codes.**

I. Introduction

THE well-known LDPC codes were first introduced in 1963 by Gallagar [1]. The rediscovery of LDPC codes by Mackey and Neal in 1995 [2] transformed the interest in LDPC codes as their BER performance asymptotically approaches to the Shannon limit [3]. LDPC codes can be classified into two categories: Regular and Irregular codes. In this paper Irregular LDPC codes are considered which can be specified by a Non-Systematic Sparse Parity-Check Matrix, H. Irregular LDPC codes show a better performance at low SNR values. Here Irregular LDPC codes are reduced to regularity as far as possible by applying the Mackay algorithm which uses Gauss Jordan Elimination method. The encoded data is then modulated and pulsed shaped for transmission. The received data is reshaped and applied to the LDPC decoder which limits the errors to the minimum, thereby giving a comparable performance with existing techniques.

II. LDPC Codes

LDPC codes are specified by a parity check matrix H having dimensions m × n, [2]. These codes are special class of linear block codes whose parity check matrix H has low density of one's (sparse). The name LDPC is resulting from the characteristic of the parity check matrix H that contains low density of 1's in comparison to the number of 0's in it. LDPC codes can also be graphically represented using Tanner Graph.

A. Graphical Representation of LDPC codes

Tanner [1] generalized LDPC codes with graphical representation known as Tanner graphs. Tanner graph also known as Bipartite Graph consisting of variable nodes (v) and check nodes (c). Tanner graph is constructed using the method that a Check node 'c' is connected to a variable node 'v' whenever element h_{ji} in H is equal to 1, as shown in Fig. 1.

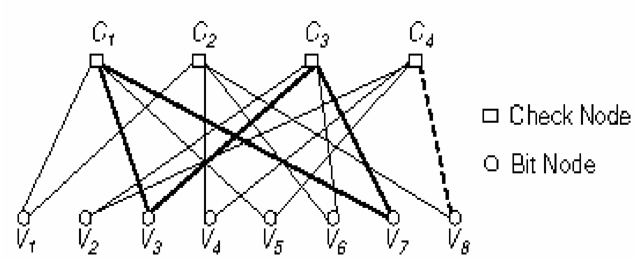

Figure 1: Tanner graph

B. Irregular LDPC codes

The irregular LDPC code can be defined by a parity check matrix H with parameters (n,j,k), where j is number of 1's in each column and k is number of 1's in each row. Also j and k would not be constant in an Irregular Matrix. However if the density of 1's in each column and the density of 1's in each row are constant then the matrix is a regular LDPC matrix. Below is an example of (6, 12) irregular parity check matrix having scarcity of 1's in it.

$$\mathbf{H} = \begin{pmatrix} 1&0&0&0&1&0&0&0&1&0&0&0 \\ 0&0&1&0&0&1&0&0&0&0&1&0 \\ 0&1&0&0&0&0&0&0&1&0&0&1 \\ 1&0&0&0&0&1&0&1&0&0&0&0 \\ 0&0&1&0&0&1&0&0&1&0&0&0 \\ 1&0&0&1&0&1&0&0&0&0&1&0 \end{pmatrix}$$

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C. Comparision of Regular and Irregular LDPC Codes

One of the basic problems with irregular LDPC is the high complexity of its encoder than that of regular LDPC encoder. But irregular LDPC codes have an advantage of showing better performance results in respect to Bit error rate (BER) for low values of SNR.

Whereas both irregular and regular LDPC codes perform well for higher codeword lengths, as code rate bounds away from 1 and this will be no longer true when codeword length becomes small, as the code rate gets close to one.

III. SYSTEM MODEL

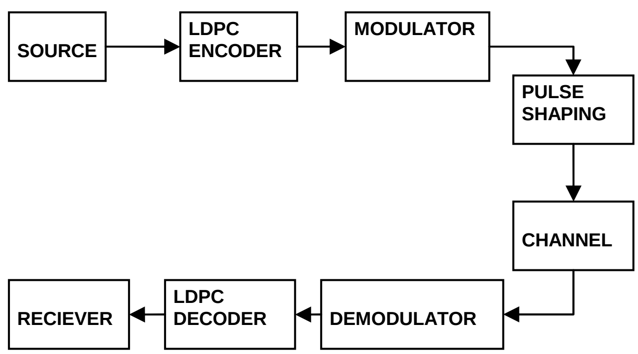

Figure 2: LDPC Simulator

In the LDPC simulator as shown in Fig 2, data is generated at source and fed into the LDPC encoder where it is encoded into codeword with the help of parity check matrix H. The encoder used here is modified Irregular LDPC coder. The encoded data is modulated using PSK and QAM modulation techniques. The modulated data is then pulse shaped using a Square root Raised Cosine Filter and then transmitted through an Additive White Gaussian Noise (AWGN) channel. The received data is reshaped, demodulated and then decoded using the LDPC decoder which further takes care of the error produced by the channel. Thus using the LDPC simulator we can transmit and receive large packets of data with better BER and spectral efficiency.

A. Generation of Parity Check Matrix

The aim is to implement an irregular parity check matrix H. First a random matrix of dimension $m\text{X}n$ is generated then an attempt is made to reduce the matrix to uniform parity check matrix using sequence of steps described by Radford Neal [2].

B. Encoding

After the generation of parity check matrix, encoded data can be produced by using the Gallagar equation

$$uH^T = 0 \qquad (1)$$

where u is the codeword and H is the parity check matrix. Assuming that message bits ‘d’ are located at the end of the codeword and the check bits ‘c’ occupy the preceding part of the codeword then

$$u = \left[c \middle| d \right] \qquad (2)$$

Also, during the generation of the parity cheek matrix, it was rearranged in the form equation

$$H' = \left[A \middle| B \right] \qquad (3)$$

where A is an identity matrix of dimension n x n . B is a matrix of dimension n x (n-m). This rearrangement of the parity check matrix H was done after the generation of parity check matrix. Thereby the data bits are then used along with the rearranged parity check matrix to generate the check bits. Using equation (2) and (3) in (1) we have

$$Ac + Bd = 0 \qquad (4)$$

$$\Rightarrow \qquad c = A^{-1}Bd \qquad (5)$$

Here A should be a non singular matrix. One of the important aspects of this technique that has to be taken into account is that the codeword c is generated using the rearranged parity matrix H’= [A|B]. The parity check matrix H cannot be used to decode the codeword since it was encoded using matrix H’. Therefore we need to rearrange the codeword according to the rearrangement of the parity check matrix done by Gauss Jordan Elimination method [2]. So as the rearranged codeword can be decoded at the receiver terminal by using the same parity check matrix H which can be encrypted at both the ends.

C. Modulation techniques and pulse shaping

Modulation is one of the basic concepts of communication which is used to load a signal onto a carrier wave that could be transmitted over a channel. A fundamental problem that often arises in the communication systems is of detecting a pulse transmitted over a channel which is degraded by channel noise. The encoded data is modulated using various techniques. BPSK and 16QAM modulation techniques are used here. The square root raised cosine filter eradicates the Inter Symbol Interference from the received data and results in an improved performance.

The impulse response of Raised cosine Pulse used for pulse shaping is given by:

$$h(t) = \sin c(\pi \frac{t}{T}) \frac{\cos(\frac{\pi\beta t}{T})}{1 - \frac{4\beta^2 t^2}{T^2}} \qquad (6)$$

where, $\beta = \dfrac{\Delta f}{(\frac{1}{2T})} = \dfrac{\Delta f}{\frac{R_S}{2}} = 2T\Delta f$

The ideal square root raised cosine filter’s frequency response has unity gain at low frequencies. Thus the bandwidth efficiency and BER of the system is improved for different modulation schemes.

D. Decoding

In this paper "sum-product decoder" [2], [3] has been used for decoding of an irregular parity check matrix of dimension m × n such that the received vector r should satisfy the condition

$$uH^T = 0$$

Here each row m=1,…, M and column n=1,…,N of H are regarded as checks and bits respectively. This set of bits and checks form a Bayesian Network in which each bit of r participates in t checks and hence every bit is the parent of t checks and verifies t_r bits. The algorithm used is suitable for binary channel model in which the noise bits are independent.

E. Decoding Algorithm

The set of bits that participate in check m are indicated by $N(m) \equiv (n : H_{mn} = 1)$ i.e. the columns which have 1's in row m of H and for each m there are t_r elements in this set. Similarly, we define the set of checks in which bit participates, $M(n) \equiv (m : H_{mn} = 1)$ and for each n there are t elements in this set. We denote a set of $(t_r - 1)$ elements participating in check m apart from bit n by $N(m)\backslash n$ and $M(n)\backslash m$ is for $(t-1)$ check apart from m check, that bit n participates in.

The algorithm has two alternating parts, in which quantities q_{mn}^x and r_{mn}^x associated with each nonzero element in the H matrix are iteratively updated. The quantity q_{mn}^x is meant to be the probability that bit n of r has a value, given the information obtained via $(M-1)$ checks apart from check m. The quantity r_{mn}^x is meant to be the probability of check being satisfied by bit n of r, if r_n is considered fixed at $x \in (0,1)$ and the other $(N-1)$ bits have a separable distribution given by the probabilities $\left(q_{mn}^x : n' \in N(m)\backslash n\right)$.

- Initialization: In this step we initialize q_{mn}^0 and r_{mn}^0 to the likelihood of x^n, f_n^0 and f_n^1.

  For the AWGN channel,

  $$f_n^1 = 1/\left(1 + e^{\frac{-2ay_n}{\sigma^2}}\right) \quad \text{and} \quad f_n^0 = 1 - f_n^1$$

  where the input to Gaussian channel is $\pm a$, $\sigma^2 = N_0/2$ is the variance of the additive noise and y_n is the soft output of the Gaussian channel.

- Horizontal Step: In this we compute the probabilities r_{mn}^0 and r_{mn}^1. For this we use the method of forward and backward passes in which products of the differences $\delta q_{mn} \equiv q_{mn}^0 - q_{mn}^1$ are computed. And also, we obtain $\delta r_{mn} \equiv r_{mn}^0 - r_{mn}^1$ from the identity: $\delta r_{mn} = \prod \delta_{mn'}^0 : n' \in N(m)\backslash n$ and then we set, $r_{mn}^0 = (1 + \delta r_{mn})/2$ and $r_{mn}^1 = (1 - \delta r_{mn})/2$.

- Vertical Step: In this step we update the value of q_{mn}^0 and r_{mn}^0 using the value of r's in previous step.

  $$q_{mn}^0 = \alpha_{mn} f_n^0 \prod r_{m'n}^0 \qquad m' \in M(n)\backslash m$$

  $$q_{mn}^1 = \alpha_{mn} f_n^1 \prod r_{m'n}^1 \qquad m' \in M(n)\backslash m$$

  Where α_{mn} is chosen such that $q_{mn}^0 + q_{mn}^1 = 1$. These products can be efficiently computed in a downward pass and an upward pass.

  The "pseudo posterior probabilities" q_n^0 and q_n^1 at this iteration can also be calculated such that

  $$q_n^0 = \alpha_n f_n^0 \prod r_{mn}^0 \quad m \in M(n)$$

  $$q_n^1 = \alpha_n f_n^1 \prod r_{mn}^1 \quad m \in M(n)$$

- Tentative Decoding: Set $r_n = 1$ if $q_n^1 > 0.5$ and $r_n = 0$ otherwise. The iteration repeats itself until $rH^T = 0$

The decoding algorithm is an iterative algorithm and it runs itself for certain number of times and halts if the checks are satisfied or it stops when maximum iterations are done with.

IV. Simulation and Results

Simulation has been done for the different code length and for various modulation techniques. Some of the results are shown here are giving better performance.

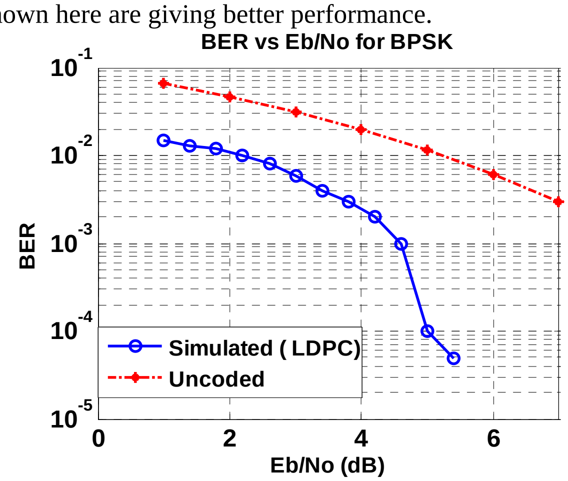

Figure 3: BER vs. Eb/No for BPSK modulation

Fig 3 shows that the BER performance for BPSK modulation over AWGN channel for the purposed technique is better than uncoded BPSK.

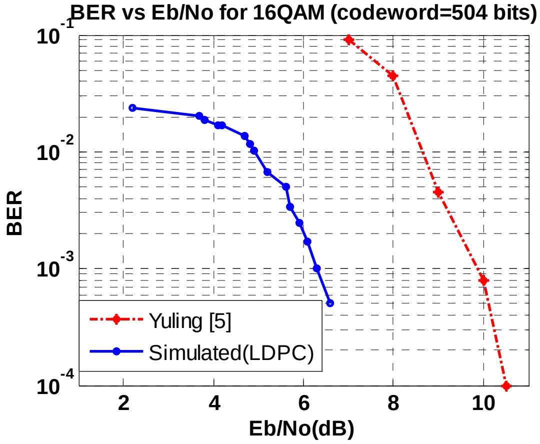

Figure 4: BER vs. Eb/No plot for 16QAM

BER performance of the purposed technique for 16-QAM modulation with AWGN channel is compared with the given result [5]. Significant improvement in the performance has been achieved even for small Eb/No as shown in Fig 4.

Comparison in Fig 5 shows that the BER performance for large codeword length (length =1008) is excelled over smaller codeword length (length=504) and can be analyzed easily.

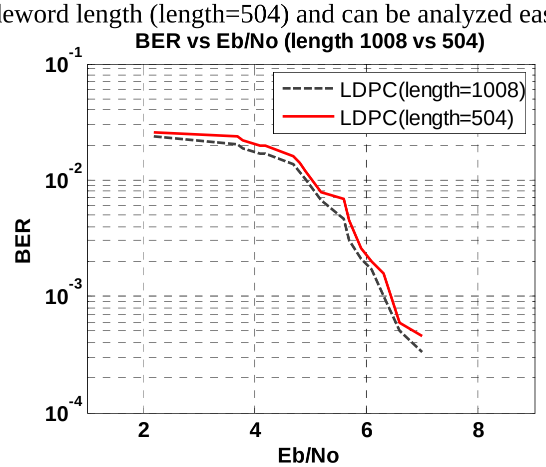

Figure 5: BER vs Eb/No for 16-QAM of different

V. Conclusion

The proposed technique gives a better BER performance for a given Eb/No as shown in results. So the performance of Irregular LDPC codes can be improved very much by attaining it towards regularity. Further, it can be concluded that using square root raised cosine filter and sum-product algorithm for decoding better results can be achieved.

Also comparative performance of 16-QAM of codeword length 1008 and 504 is done, justifying that BER decreases as codeword length increases.

VI. Acknowledgment

Authors are very thankful to Jaypee Institute of Information Technology University for providing this opportunity to do this work.

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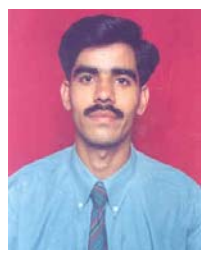
VIII. Biographies



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