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Optimization of Pilot Symbol Spacing and Receive Antenna Diversity in Adaptive Pilot Symbol Assisted Modulation for Rayleigh Fading Channel

Kuldeep Yadav, Neetu Singh and M.C. Srivastava

***Abstract*— Channel state prediction is required in adaptive modulation to attain highly spectral efficient transmission. In this paper, study has been carried out for optimized pilot symbol spacing in adaptive Pilot Symbol Assisted Modulation (PSAM). This technique is made adaptive by varying the pilot symbol spacing in the transmitted signal. Also, receive antenna diversity is used to account for the effect of change in carrier signal to noise ratio (CSNR) with the help of a low capacity feedback channel. Optimization of pilot symbol spacing is done using Sequential Quadratic Programming (SQP) algorithm. For an optimized value of pilot symbol spacing, optimum number of antennas to be used is calculated to maximize average spectral efficiency (ASE). The results of the proposed technique are compared with that of PSAM and adaptive PSAM techniques. A considerable improvement in terms of Mean Square Error (MSE), Bit Error rate (BER) and ASE has been achieved for an optimal value of pilot spacing and number of receiver antennas.**

Keywords —Channel State Prediction, Adaptive PSAM, Receive Antenna Diversity, SQP, Optimization, MSE, BER, ASE.

I. INTRODUCTION

The demand for reliable and high data rate transmission is growing day by day. Since, both spectrum and power usage are strictly regulated and bandwidth is also scarce, there is a need to transmit maximum information bits per second per unit bandwidth necessary on an average while maintaining a certain quality. Adaptive Modulation is one way of realizing such schemes, which completely depend on the knowledge of channel state information at the transmitter side. Thus, it is a great task to predict or estimate the state of channel at any given instant of time.

In PSAM technique [1], transmitter periodically inserts known symbols in transmitted frame, from which the receiver [1] derives its amplitude and phase reference. Further this technique is made adaptive PSAM by varying the pilot symbol spacing adaptively with respect to the CSNR with the help of a low capacity feedback channel. Based on this channel prediction transmitter dynamically adapt rate and modulation to a mode which maximizes the ASE while maintaining the instantaneous BER below some predefined target value, BER_0.

The outline of this paper is described in four sections. In section II, channel modeling and channel prediction of the proposed system model, based on statistical measurements made specifically for an intended communication system or spectrum allocation [2, 7] are described. The implementation of PSAM that characterizes the fading channel employs LRP algorithm. This algorithm computes the Minimum Mean Squared Error (MMSE) estimate of a future fading coefficient sample based on a number of past observations [6, 7 and 11].

In Section III, the proposed Adaptive PSAM technique is described along with optimization of pilot symbol spacing and receiver antenna diversity to maximize the spectral efficiency. In the proposed scheme, the PSAM is first optimized in terms of ASE as a function of pilot symbol spacing and order of the channel predictor. Next optimal number of antennas with large predicted CSNR are selected using threshold combining.

Finally, simulation results such as MSE [dB] vs. CSNR [dB], ASE vs. CSNR [dB], BER vs. CSNR [dB], for comparison between existing PSAM and proposed APSAM techniques are discussed in section IV.

II. PROPOSED APSAM SYSTEM MODEL

A. Channel Modeling

In most of the wireless systems operating in urban areas there is no direct line of sight (LOS) path between the transmitter and the receiver. Also, the presence of various disturbances in the path causes different type of severe losses. This results into multipath reflections of transmitted wave and interaction between the reflected waves causes multipath fading at receiver. Thus, the amplitude and phase of the received wave may vary widely. This variation depends upon the distribution of the intensity, relative propagation time of the waves and the bandwidth of the transmitted signal.

The existence of multiple propagation paths can be assumed between transmitter and receiver for a typical terrestrial channel. With each transmission path propagation delay and an attenuation factor are associated, which are

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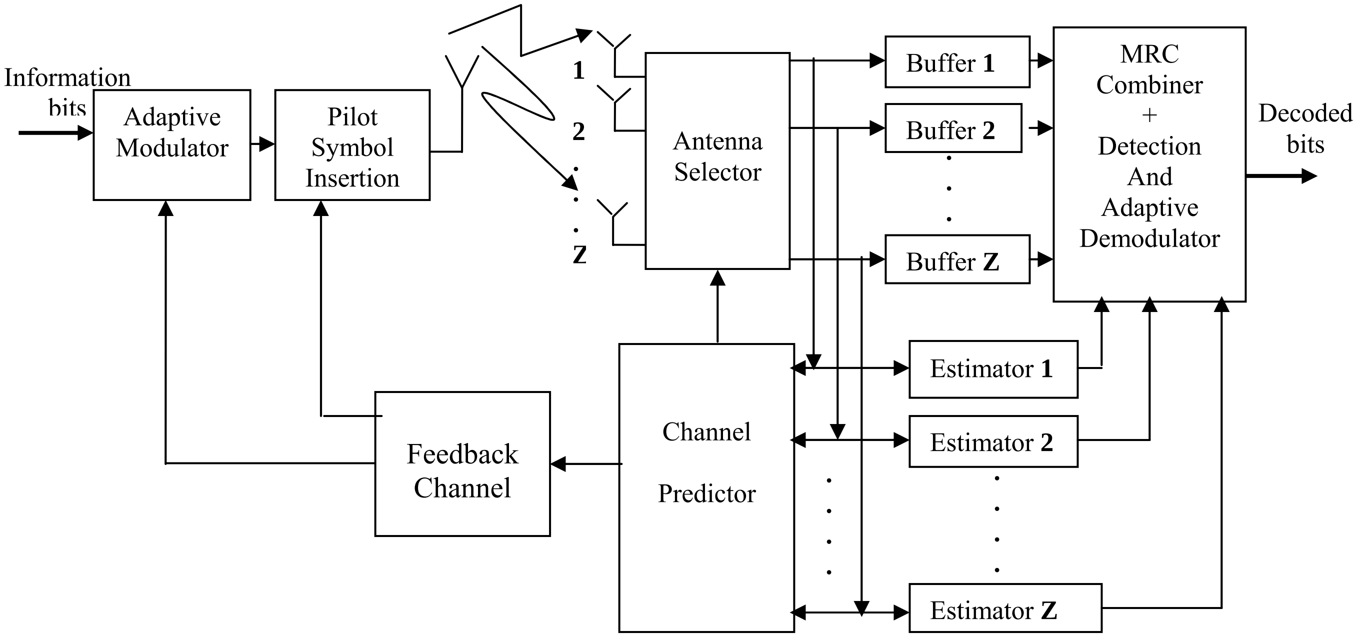

Figure 1. Adaptive Coded Modulation System with APSAM

usually time-varying due to changes in propagation conditions, resulting primarily from transceiver mobility. In addition, there is also a Doppler shift in the channel.

Let the information signal transmitted by base station be

$$s(t) = \mathrm{Re}\left\{x(t).e^{j2\pi f_c t}\right\} \qquad (1)$$

where $x(t)$ is complex lowpass signal, f_c is carrier frequency.

Following [11], the received complex baseband signal in such a channel at Zth antenna in the absence of additive noise is given by

$$y_Z(t) = \sum_{m=1}^{M} \alpha_m(t) e^{-j[\phi(t)]} u(t - \tau_m(t)) \qquad (2)$$

where $\phi(t) = 2\pi[(f_c \pm f_{D,m}(t))\tau_m(t) - f_{D,m}(t)t]$,

M corresponds to total number of paths, $\alpha_m(t)$ and $\tau_m(t)$ are respectively the path attenuation and the propagation delay for the signal received on the mth path.

The multipath fading channel can be modeled as a time-varying linear filter with impulse response $h_Z(\tau,t)$ given by

$$h_Z(\tau,t) = \sum_{m=1}^{M} \alpha_m(t) e^{-j[\phi(t)]} \delta(t - \tau_m(t)) \qquad (3)$$

Assuming a large number of paths between the transmitter and the receiver, and that the associated attenuations per path are independent and identically distributed, impulse response $h(\tau,t)$ can be modeled by a complex- valued Gaussian stochastic process. If the received signal $y(t)$ has only diffused multipath components, $h(\tau,t)$ has a Rayleigh distribution and hence the channel is called a Rayleigh channel.

B. Channel Prediction

The proposed PSAM model is made adaptive to changing wireless environment conditions, specifically time varying channel as shown in Fig. 1. The number of pilots, the power allocated for pilot symbols and the locations of these pilots in the data stream all affect the system performance measured by the reliable transmission rate, average spectral efficiency (ASE), bit error rate (BER), or the mean square error of the estimator.

After the insertion of pilot symbols, the received signal at any time, k for Zth antenna can be expressed as

$$y_z(k) = h_z(k)x(k) + n_z(k) \qquad (4)$$

where k = mL, ($m \in \{0,1,2..\}, L \in \{1,2,3..\}$). It is assumed that all the pilot symbols have the same absolute value, $|x(mL)| = a_p$.

Assuming Z statistically independent branches of antenna, maximal ratio combining (MRC) can be implemented at the receiver. Following Oien et all [3] the ML optimal estimate of set of memoryless static channel parameters $h(n-kL)$ based on received observations $\boldsymbol{y}$ for any branch of antenna can be expressed as

$$\widetilde{h}(n-kL) = h(n-kL) + \frac{N(n-kL)}{a_p} \tag{5}$$

The second term in (5) corresponds to the noise signal received on one channel divided by the known pilot symbol value.

The prediction of channel j symbols ahead in time, i.e. n + j of the last received pilot symbol can be made from K memoryless ML estimates. Further, $\hat{\boldsymbol{h}}(\boldsymbol{n}+\boldsymbol{j})$ the linear predictor of order K can be written in the form

$$\hat{h}(n+j) = \sum_{k=0}^{K-1} f_j(k)\widetilde{h}(n-kL) = f_j^T \widetilde{h} \tag{6}$$

where $f_j^T = [f_j(0), f_j(1), \ldots f_j(K-1)]$ the predictor filter coefficient vector corresponds to delay j.

Similarly MAP-optimal prediction filter coefficient vector for the complex fading amplitude on a Rayleigh channel can be expressed as

$$f_{j,MAP}^T = r_j^T (R + \frac{1}{\bar{\gamma}} I)^{-1} \tag{7}$$

where the j time index denotes the delay counted in number of channel symbols.

With power transmitted P , the predicted overall CSNR $\hat{\gamma} = \dfrac{\alpha^2 P}{N_0 B}$ is gamma-distributed with expectation

$$E[\hat{\gamma}] = \bar{\hat{\gamma}} = r\bar{\gamma} \tag{8}$$

The parameter r can be viewed as the ratio between the expectation of the predicted and the true channel CSNR.

The overall received CSNR, and gain is given by

$$\gamma(k) = \sum_{z=1}^{Z} \gamma_z(k) \tag{9}$$

$$\hat{\alpha}^2 = \sum_{z=1}^{Z} \alpha_z^2 \tag{10}$$

where α is the absolute value of fading envelope.

III. Adaption Techniques In APSAM

Two aspects of optimization- Pilot symbol spacing and Receive antenna diversity employed in the Adaptive PSAM are discussed in this section.

To maximize spectral efficiency, pilot symbol spacing L and predictor order K can be optimized, which in turns result into reduced MSE and also improved BER. In addition, effect of diversity is also introduced to get better results in terms of spectral efficiency and BER.

With the assumption that channel characteristic does not vary between two successive pilot symbols, γ remains unchanged in the same CSNR. The BER (averaged over all codes and all CSNRs) used in the model is given as the average number of bits in error, divided by the average number of bits per symbol transmitted, i.e. the average spectral efficiency (ASE) [5]:

$$BER = \frac{\sum_{n=1}^{N} R_n BER_n}{\sum_{n=1}^{N} R_n P_n} \tag{11}$$

$$R_n = (\log_2(M_n) - \frac{1}{D})\frac{L-1}{L} \tag{12}$$

where R_n is the information rate of nth code when two-dimensional $(D \in \square)$ trellis codes are used, BER_n is the average BER experienced when nth code is used and P_n is the probability that the nth code will be used such that the predicted CSNR falls in the interval $[\gamma_n, \gamma_{n+1})$. The choice of nth code is based on the idea that the CSNR is $\hat{\gamma}$, while it is actually γ .

In (11), the value of n can also be zero, corresponding to the CSNR interval $[0, \gamma_1)$ and only strictly positive values can be used. When the CSNR is smaller than γ_1, no information will be sent. Following Dong et all [8] the average BER for nth code can be written as

$$BER_n = \int_{\gamma_n}^{\gamma_{n+1}} \int_{0}^{\infty} BER_n(\gamma/\hat{\gamma}) p(\gamma,\hat{\gamma}) d\gamma d\hat{\gamma} \tag{13}$$

where n = 1, 2 ….N and $BER_n(\gamma/\hat{\gamma})$ is the BER experienced when nth code is applied.

A. *Pilot symbol spacing*

In PSAM, pilot symbol spacing is selected according to some predefined sets of parameters and remains fixed whereas channel parameters keep on changing. If small pilot spacing is selected then for a slow varying channel and with high CSNR, fixed pilot spacing results into wastage of bandwidth in terms of low channel capacity and poor spectral efficiency and vice-versa. In order to overcome these disadvantages, pilot symbol spacing is varied in the proposed scheme according to the change in channel conditions.

In (12), the change in pilot symbol spacing L will also change the value of R_n. Also the value of BER depends upon the change in R_n and hence the changes in pilot symbol spacing L affect the value of BER as given in equation (11).

Following [9, 10] average spectral efficiency can be expressed as:

$$ASE = \frac{L-1}{L}\left[e^{-\frac{\hat{\gamma}_1}{r\gamma}} \left(\log_2(M_n) - \frac{1}{2} \right) + \sum_{n=2}^{N} e^{\left[-\frac{\hat{\gamma}_n}{r\gamma} \right]} \log_2\left(\frac{M_n}{M_{n-1}} \right) \right] \tag{14}$$

where M_n is the number of points in the symbol constellation used by the trellis code.

The value of pilot symbol spacing lies between [2, 3, 4… $L_{\max}$],

where $$L_{\max} = \frac{B}{2 f_d} \tag{15}$$

The parameters used to decide the pilot symbol spacing depend on the value of predicted CSNR. The change in pilot symbol spacing affects the error between true CSNR and the predicted CSNR. In PSAM, when a state of channel at any particular instant of time is predicted at the receiver, it derives its reference from pilot symbols and finds out whether there is any change in interval of CSNR. If there is any change in interval of CSNR, pilot symbol spacing is changed accordingly and CSI (Channel State Information) is fed back to the transmitter side by a low capacity feedback channel. This results in effect the PSAM technique becoming adaptive.

To find optimum pilot spacing, ASE is maximized by using SQP (Sequential Quadratic Programming) algorithm. This method solves the nonlinear problem directly rather than converting it into a sequence of unconstrained minimization or maximization problem.

max ASE (L, K) (16)

Subject to $2 \le L \le \frac{B}{2 f_d}$

B. *Receive Antenna Diversity*

To mitigate the effects of fading diversity is considered as an important tool. In this paper, space diversity (receive antenna diversity) is used.

In the proposed system Z receiver antennas are used. The MRC technique is used to combine the signals and it is assumed that the branches are mutually uncorrelated. After finding the optimum value of L for every value of CSNR at which ASE is maximized and with this optimized value of L, ASE is further maximized by selecting the optimum number of receiver antennas. Optimal number of antenna branches means number of branches that will be sufficient for estimation and prediction.

The modified expression for ASE can be written as:

$$ASE = \frac{L-1}{L}\left(\log_2(M_n) - \frac{1}{Di} \right)\left[\Gamma\left(Z, \frac{\gamma_n}{r\overline{\gamma}_z} \right) - \Gamma\left(Z, \frac{\gamma_{n+1}}{r\overline{\gamma}_z} \right) \right] \tag{17}$$

max ASE (L, K,Z) (18)

The SQP algorithm is again used for solving the optimization problem. To find out the branches with largest CSNR which will be best for estimation, principle of threshold combining is used.

IV. SIMULATED RESULTS

In this work simulation has been carried out for various applications in wireless communication. The parameters taken for simulation are: Carrier frequency = 2.4 GHz, Sampling frequency = 10 kHz and 50 kHz, Delay considered = 1ms for Doppler frequency of 200 Hz, vehicle velocity v =108 km/h and CSNR = 0 dB to 30 dB.

In Fig 2, MSE [dB] vs SNR per channel or CSNR in dB are plotted. A comparison of simulated results for proposed APSAM model and existing PSAM technique show a significant improvement in MSE for optimized pilot symbol spacing. For the proposed APSAM model the data for MSE simulation are: Pilot symbol to be predicted ahead = 1, Doppler shift = 200Hz, Sampling frequency = 100 KHz and CSNR = 0 to 20 dB, pilot symbol spacing chosen for PSAM is 50.

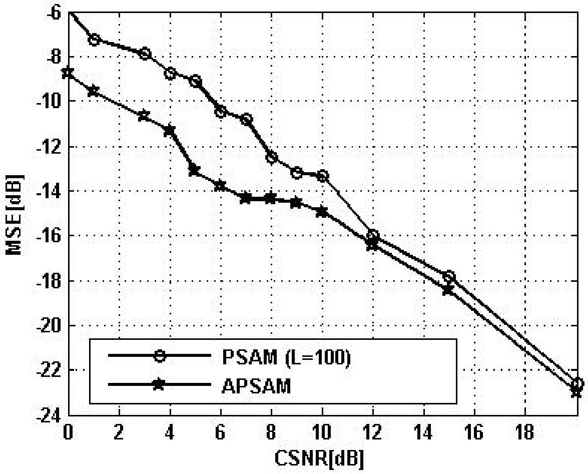

Figure 2. MSE [dB] vs. CSNR [dB]

In Fig 3, ASE vs. CSNR [dB] is compared for PSAM and APSAM technique which shows significant improvement in ASE for small values of CSNR. Here, Improvement in ASE has been shown for optimum L for every interval of CSNR only and number of receiver antennas taken as 1.

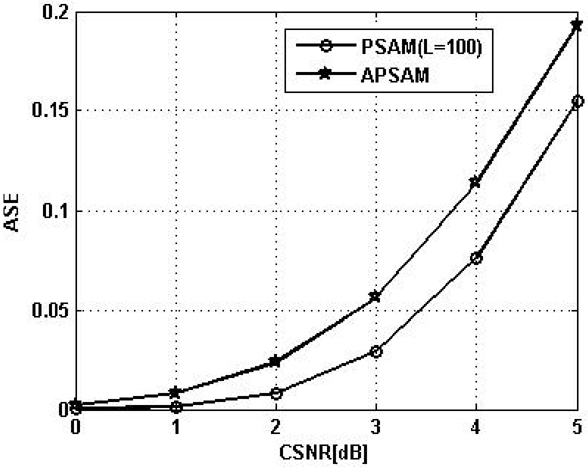

Figure 3. ASE [bps/Hz] vs. CSNR [dB]

Fig 4 shows the effect of space diversity on ASE for different number of antennas with MRC employed at the receiver. As number of receiver antennas increases from 1 to 12, it can be seen that ASE is increased from almost 0 to 1 for CSNR = 0dB and from 7 to 8.5 dB for CSNR =30dB. Also, it is found that for CSNR = 25 dB, ASE is maximum for 12 receiver antennas and for CSNR = 30dB, ASE is maximum for 8 receiver antennas.

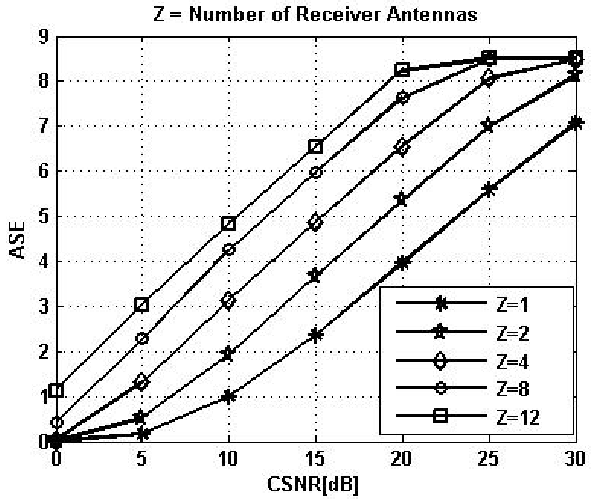

Figure 4. ASE [bps/Hz] vs. CSNR [dB]

The results of the proposed scheme are compared with the scheme proposed by Cai et all [9]. The system parameters are chosen as: Sampling frequency =200 kHz, and pilot symbol spacing is taken as 10[9]. In Fig 5, Comparison of ASE for optimal L and optimum number of receiver antennas ***Z*** is shown.

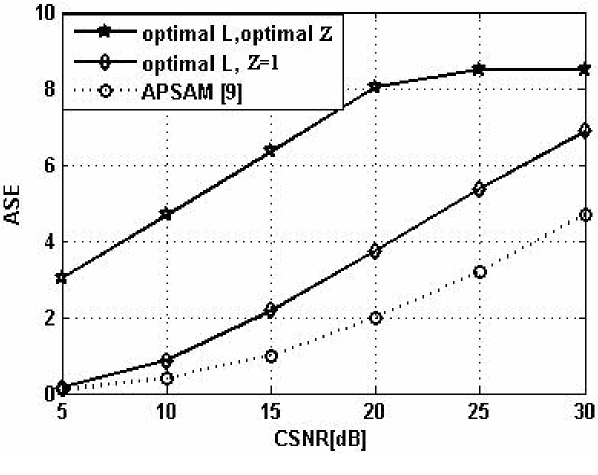

Figure 5. ASE [bps/Hz] vs. CSNR [dB]

In Fig 6, with target BER_0 $=10^{-5}$, results of BER are shown for optimal L with optimal number of receiver antennas and compared with BER results of APSAM [9]. The value of BER decreases very rapidly for higher value of CSNR.

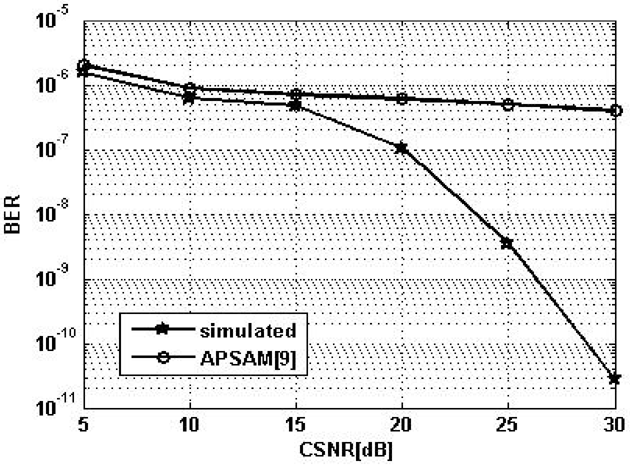

Figure 6. BER vs. CSNR [dB]

V. CONCLUSION

In this work Adaptive PSAM technique has been analyzed and new scheme is proposed which gives significant improvement in spectral efficiency and bit error rate. The spacing between two consecutive pilot symbols has been optimized to maximize the spectral efficiency.

As shown in the results it is found that using the receive antenna diversity further improvement in spectral efficiency can be achieved. This technique can achieve maximum value of ASE for a set of finite number of codes as CSNR grows. Also, after optimization of the diversity better results can be achieved in terms of ASE even for the lower value of CSNR.

Simulated results show that the proposed scheme work well and give better results in terms of BER even for the small value of CSNR.

VI. ACKNOWLEDGMENT

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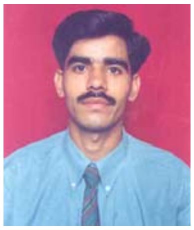

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