Thin-Plate Splines in Registration of Various Deformed Images

Asmita A Moghe, Indrajeet Singh, Jyoti Singhai and S.C Shrivastava

***Abstract*-- An image is said to be elastic in nature when deformations in the image can be expressed in terms of similarity features or forces deforming it and forces resisting deformation by the elastic model. An analysis of registration of such images (cameraman and abdominal computed tomography) has been done to assess global, or local geometric deformations and non-rigid deformations. Registration of images based on feature point selection has been readily employed using Thin-Plate spline interpolation Accurate selection of control points plays a key role in increasing the correlation between the two images. Scaling and translation parameters are close to actual values as precise selection of control points is done. Accurate selection of control points becomes vital for improving diagnostic value in case of registration of medical images.**

***Index Terms*—Correlation, image matching, image registration, spline functions, strain**

I. Introduction

IMAGE registration refers to matching of two images or mapping the coordinates of the two images. This is required in applications related to remote sensing, medical imaging, finger print detection, facial recognition, and fusion just to name a few [1]. The need to match or register two images arises when the two images to be matched are acquired from different scanners, at different points of time or from different viewpoint. In these conditions images can undergo translation, rotation or scaling. Such deformations in images are termed as rigid whereas those undergoing shearing etc additionally are termed as non-rigid deformations. The latter may also occur due to deformations within the image itself as in case of lesions in involuntary organs within the image. A survey on image registration methods indicates a constant growth in research in this area since past twenty years [2] with a lot of work being done in the area of elastic registration and in high end modalities like those of PET (Positron Emission Tomography), SPECT (Single Photon Emission Computed Tomography). A relatively detailed information on various image registration methods as applied to rigid and non rigid registration can be found in [3]. It spells out the need to lay emphasis on feature-based methods with modality invariant features being chosen. It also puts forth the need to develop registration methods with minimum computational time, better registration accuracy and use of a combination of registration methods to acquire the above. Several methods that combine intensity based registration and feature-based registration have been employed that are either operator dependent, semi automatic or fully automatic. [4], [5] and [8] are hierarchical methods, [6],[7] and [9] combine intensity based method and landmark based methods to register the images.

In the process of registration, images to be matched or registered include a reference and the test image. The test image is also referred to as the input or target image. The target image is usually the one that has undergone geometric transformations like translation, rotation, scaling or shear globally or locally or may be a combination of both. Image is then said to have undergone deformation. So to register the two images the test image (deformed image) is suitably transformed. The coordinates of transformed points are then interpolated or rounded. The intensities at these points are then interpolated into the reference image. This brings the test image as close as possible to the reference image and is then said to be registered.

No material body is perfect and is therefore liable to some form of deformation. A body is said to be rigid if relative positions between particles remain unchanged. However if relative positions between particles get displaced we say that it has undergone deformation. These deformations can be local or global, small or large, elastic or plastic.

In our experiment, an analysis of deformations in some images is carried out to bring out the subtle deformations in images that may be differing slightly from each other in order to improve the analysis or diagnostic value (in case of medical images). The corners of the images and invariant points that are prominent in the image known as control points or feature points, are selected manually and are interpolated using Thin Plate Spline (TPS) transfer function. Thin Plate Splines are essentially chosen as they smoothly interpolate the scattered data for small deformations.

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II. ELASTIC REGISTRATION

A. ElasticDeformation

As mentioned above when the distances between the various points in a material body get altered, the body is said to have undergone deformation and when these deformations cannot be expressed linearly the deformations are said to be non-linear or non-rigid or elastic deformations. These deformations can then be modeled using polynomials [1] or by use of splines. When a body is said to be elastically deformed it is due to some external forces acting on the body that are responsible for deforming it. Also there will be some internal forces resisting this deformation. These forces counterbalance each other till a minimum energy state is reached. This is minimization of the cost function

Cost = Cost(deformation) – Cost(Similarity)

This is dealt in [10]. The Bending energy matrix gives this minimum energy state, which should be a minimum. Thus to carry out elastic registration, the test image is considered as an elastic body where external forces acting on it are the similarity features identified by selecting the suitable control points in the two images and the internal forces are determined by the Thin Plate Spline model. [11]

B. Thin-Plate Spline Model

A deformed surface of irregularly spaced or scattered points is given by $z(x,y)$ where r is the distance $\sqrt{x^2+y^2}$ from cartesian origin with negative sign being used for convenience. $U(r)$ satisfies the bi-harmonic equation given by

$$\Delta^2 U = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) U \,\alpha\, \delta_{(0,0)} \qquad (1)$$

where the function $U(r)$ defines the surface $z(x,y)$ given as

$$U(r)\ z(x,y) = -U(r) = -r^2 \log(r^2) \qquad (2)$$

Fundamental solution of bi-harmonic equation $\Delta^2 U = 0$ gives the equation of a Thin –plate spline lofted as a function $z(x,y)$ above (x,y) plane. Depending on the constraints, the steel plate when subjected to slight bending, bending energy is proportional to

$$z(x,y) = \iint_{R^2} ((\partial^2 z/\partial x^2)^2 + 2(\partial^2 z/\partial x \partial y)^2 + (\partial^2 z/\partial y^2)^2)) dxdy \quad (3)$$

at that point and minimizes

$$z(x,y) = \iint_{R^2} ((\partial^2 z/\partial x^2)^2 + 2(\partial^2 z/\partial x \partial y)^2 + (\partial^2 z/\partial y^2)^2)) dxdy$$

To determine the bending energy, TPS interpolation function is determined from the two sets of control points in the reference and target image. If P1 (x,y)…..Pn(x,y) be n points in reference image and Q1(x,y)…..Qn(x,y) be corresponding points in the target image then the TPS interpolating function is given by the equation

$$f(x,y) = a_1 + a_x x + a_y y + \sum_1^n w_i U\left(\left|P_i - (x,y)\right|\right) \qquad (4)$$

The coefficients a_1, a_x, a_y and weights w_i can be found by determining the matrices K,P,L and $L^{-1}Y$ as below.

$$K = \begin{bmatrix} 0 & U(r_{12}) & .... & U(r_{1n}) \\ U(r_{21}) & 0 & ..... & U(r_{2n}) \\ .... & .... & 0 & .... \\ U(r_{n1}) & U(r_{n2}) & .... & 0 \end{bmatrix}_{nxn}$$

$$P = \begin{pmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ ... & ... & ... \\ 1 & x_n & y_n \end{pmatrix}_{3xn}$$

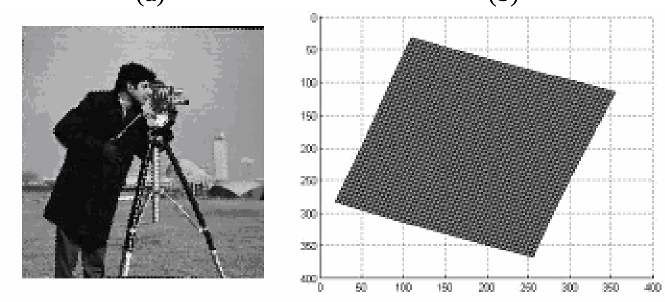
$$L = \begin{pmatrix} K & P \\ L^T & O \end{pmatrix}_{(n+3)x(n+3)}$$

Here O is a 3 x 3 zeroes matrix.Y is a column vector having length n+3 containing corresponding n control points in target image and 3 zeroes. Weights w_i and coefficients a_i in equation (4) can be obtained from $L^{-1}Y$ The resulting function f (x,y) will map each point Q_i (x,y) in target image to it's to it's homologue P_i (x,y) in reference image with minimum bending according to the bending energy according to (3).

III. EXPERIMENTAL ANALYSIS

'Cameraman' image that is used as one of the reference image for first three parts of the experiment was deformed by providing geometric transformations (rigid,affine) and polynomial transformation. The other reference image is the abdominal CT (Computed Tomography) image for the fourth part of experiment with cyst having unknown deformations (non–rigid). In each of the cases control points were selected in original (reference) image and similar points were selected in the target image and the images were registered using the Thin- Plate splines to interpolate the scattered data. Response of Thin Plate spline interpolation to deformed images was analyzed on the basis of correlation coefficient, rotation and translation parameters. Rotation angle was 60 degree, translation along x and y direction was by 2 pixels, scale factor of 0.8. Fig. 1(a) and (b) give Cameraman reference and deformed image having rigid deformation (translation and rotation) and Fig. 1(c) gives the corresponding registered image when Fig. 1(b) is registered to Fig. 1(c) using Thin Plate Spline as interpolant. Fig. 1(d) gives the corresponding warped or deformed grid. The correlation coefficient with successively increasing number of control points is as given in the Table. I

(a) (b)



(c) (d)

Fig. 1 (a) Reference image (b) Deformed image (c) Registered image (d) Deformed grid.

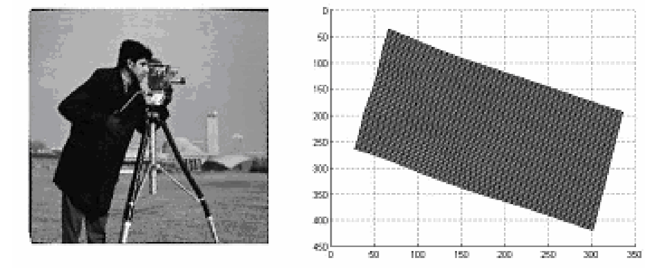
TABLE. I
CORRELATION COEFFICIENT, ROTATION AND SCALING IN CASE OF GEOMETRICALLY DEFORMED IMAGE (TRANSLATION, ROTATION AND SCALING)

No. of Control points	Correlation Coefficient	Rotation	Scaling S_x S_y
9	0.9044	25.448	0.8151 0.7941
13	0.9299	22.5866	0.8117 0.7918
15	0.9280	22.5866	0.8117 0.7918

The same experiment was repeated for cameraman image that underwent affine transformation. There is seen to be an improvement in correlation coefficient. The reference image and affine transformed reference image called deformed image are as shown in Fig. 2(a) and Fig.2 (b).

(a) (b)

Fig. 2 (a) Reference image (b) Deformed image



(c) (d)

Fig. 2 (c) Registered image (d) Deformed grid

The corresponding registered image is as shown in Fig. 2(c) and the deformed is as shown in Fig. 2(d). The correlation coefficient, rotation and translation parameters thus obtained for this experiment for affine transformed image that was registered is as given in Table. II.

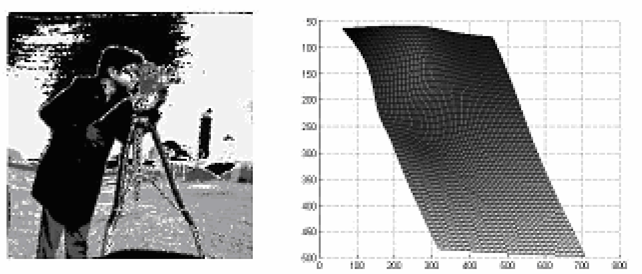
TABLE. II
CORRELATION COEFFICIENT, ROTATION AND SCALING IN CASE OF GEOMETRICALLY DEFORMED IMAGE (TRANSLATION, ROTATION, SCALING AND SHEAR)

No. of Control points	Correlation Coefficient	Rotation	Scaling S_x S_y
10	0.5252	66.1339	0.9293 0 .6852
13	0.8337	67.8396	0.9698 0.6634
14	0.8345	68.9735	0.9678 0.6669

The same cameraman reference image was further deformed polynomially and when registered similarly the results are as given in Fig. 3(a)-3(d). The deformations here are more. It is seen that correlation coefficient improves gradually as control points are taken for the points where changes or variations can be precisely distinguished. Also greater numbers of control points are required to accurately register these images. Fig. 3(a) and 3(b) represent the corresponding reference and polynomially deformed image.

Fig. 3. (a) Reference image (b) Deformed image (rotated initially then polynomially warped)

The image obtained after registration of polynomially deformed cameraman image to the reference cameraman image is as shown in Fig. 3(c) and and the corresponding deformed or warped grid is as shown in Fig. 3(d)



(c) (d)

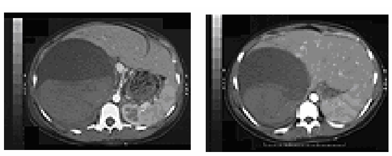
Fig. 3. (c) Registered image (d) Deformed grid.

For a set of varying number of control points the corresponding correlation coefficient, rotation and translation parameters so obtained are as given Table III.

TABLE. III
CORRELATION COEFFICIENT, ROTATION AND SCALING IN CASE OF POLYNOMIALLY DEFORMED IMAGE

No. of Control points	Correlation Coefficient	Rotation	Scaling S_x S_y
12	0.2918	28.41	1.0272 0.7636
18	0.3572	35.5292	1.0430 0.7057
22	0.4495	20.2486	1.3243 0.6163

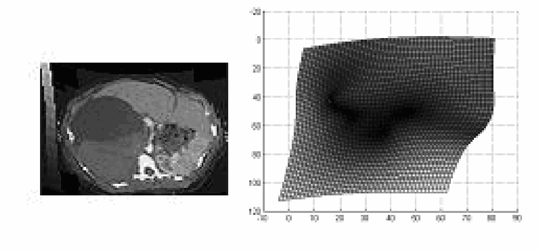
For the fourth part of the experiment the images on which the experiment was done are the abdominal CT images of a female having cyst .The images of the abdomen as shown in Fig. 4 (a) and 4(b) are taken before and after the delivery at 35 weeks pregnancy .The cyst that was earlier suspected to be in liver was found to be in the right adrenal gland.



(a) (b)

Fig. 4 (a) Reference image (b) Deformed image (unknown deformation)

Applying elastic registration to the two abdominal CT images it is seen that registration accuracy is seen to be much better giving a relatively high correlation coefficient. Similar results can also be obtained by interpolating the corresponding X component and Y component mapping functions as given by [12]. Use of Multiquadric, Weighted mean instead of logarithmic functions can be suitable but only according to specific application [13]. For all other non-rigid deformations Thin Plate Splines give a smooth surface. Applying segmentation to images and selecting control points automatically is the most sought after method these days however manual selection of control points still provides the fine access to deformations. Corresponding registered image and deformed grid for the abdominal CT images are as shown in Fig. 4 (c) and Fig. 4(d) respectively.



(c) (d)

Fig. 4 (c) Registered image (d) Deformed grid.

Table. IV shows that correlation coefficient for the two CT images gradually improves with increase in control points provided a proper choice of control points is made

TABLE. IV
CORRELATION COEFFICIENT, ROTATION AND SCALING IN CASE OF POLYNOMIALLY DEFORMED IMAGE

No. of Control points	Correlation Coefficient	Rotation	Scaling S_x S_y
7	0.3103	41.3316	1.957 1.0205
11	0.3459	75.5166	1.0943 0.9072
16	0.6193	60.3962	1.1140 0.8697

IV. CONCLUSION

Response of different deformed images when elastically registered was observed for all the four cases. It was found that registration accuracy in case of global geometric differences was relatively more than in case of local geometric differences as in Fig.3. However for the type of non-rigid deformation as in Fig.4 registration accuracy improves. Registration accuracy does not vary linearly with increase in number of control points. Increase in number of control points necessarily increases the registration time. Precise selection and proper choice of control points plays a key role in determining registration accuracy. Scale factors and rotation angles are more close to actual values with proper selection of control points as suggested by the increase in correlation coefficient. To improve registration accuracy, either some other interpolation function can be chosen that does not vary logarithmically or hierarchical initial registration can be done prior to elastically registering the images.

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VI. BIOGRAPHIES

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