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A Study on various Histogram Equalization Techniques to Preserve the Brightness for Gray Scale and Color Images

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***Abstract:*-Histogram equalization (HE) works well on single-channel images for contrast enhancement. However, the technique used is ineffective on multiple channel images. So, it is not suitable for consumer electronic products, where preserving the original brightness is necessary in order not to introduce unnecessary visual deterioration. Bi-histogram equalization (BHE) has been developed and it is analyzed mathematically. BHE separates the input image's histogram into two, based on its mean before equalizing them independently so that it can preserve the original brightness up to certain extends. Recursive Mean-Separate Histogram Equalization (RMSHE) is another technique to provide better and scalable brightness preservation for gray scale and color images. While the separation is done only once in BHE, RMSHE performs the separation recursively based on their respective mean. It is analyzed mathematically that the output images mean brightness will converge to the input images mean brightness as the number of recursive mean separation increases. The recursive nature of RMSHE also allows scalable brightness preservation, which is very useful in consumer electronics. Finally a comparative study was made to analyse all the above methods using gray scale and color images.**

***Index Terms:* Bi-histogram equalization, histogram equalization, scalable brightness preservation, recursive mean-separate.**

I. INTRODUCTION

HISTOGRAM equalization (HE) is a very popular technique for enhancing the contrast of an image. Its basic idea lies on mapping the gray levels based on the probability distribution of the input gray levels. It flattens and stretches the dynamics range of the images histogram and resulting in overall contrast improvement, HE has been applied in various fields such as medical image processing and radar image processing [1] & [2].

But, HE is not commonly used in consumer electronics such as TV because it may significantly change the brightness of an input image and cause undesirable artifacts. In theory, it can be shown that the mean brightness of the histogram-equalized image is always the middle gray level regardless of the input mean. This is not a desirable property in some applications where brightness preservation is necessary.

Mean preserving Bi-histogram equalization (BHE) has been proposed to overcome the above mentioned problems. BHE firstly separate the input images histogram into two based on its mean; one having range from minimum gray level to mean and the other ranges from mean to the maximum gray level. Next, it equalizes the two histograms independently. It has been analyzed both mathematically and experimentally .This technique is capable to preserve the original brightness to a certain extends [2].

There are still cases that are not handled well by BHE. These images require higher degree of brightness preservation to avoid annoying artifacts. Therefore, this paper proposes a generalization of BHE to overcome such limitation and provide not only better but also scalable brightness preservation. BHE separates the input images histogram into two based on its mean before equalizing them independently. While the separation is done only once in BHE, this paper proposes to perform the separation recursively; separate each new histogram further based on their respective means. It has been analyzed mathematically that the output images mean brightness would converge to the input images mean brightness as the number of recursive mean separations increases. Besides, its recursive nature also implies scalable preservation, which is very useful in consumer electronic products. The generalization of BHE, namely – Recursive Mean Separate Histogram Equalization (RMSHE) will be presented with mathematical analysis [7].

II. HISTOGRAM EQUAIZATION

This section covers the details regarding HE and BHE well as their mathematical analysis in brightness preservation. It is basically a reprint [2] and [4].

A. Typical Histogram Equalization

For a given image X, the probability density function $P(X_k)$ is defined as

$$P(X_k) = n^k / n \qquad (1)$$

For k=0,1,...,L-1, where n^k represents the number of times that the level X_k appears in the input image X and 'n' is the total number of samples in the input image. Note that $P(X_k)$ is associated with the histogram of the input image which represents the number of pixels that have a specific intensity X_k. Based on the probability density function , the cumulative density function is defined as

$$\sum_{j=0}^{k} P(X_j) \qquad (2)$$

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Where $X_k = X$, for k=0,1,…,L-1. Note that $C(x_{L-1}) = 1$ by definition. HE is a scheme that maps the input image into the entire dynamic range, (X_0, X_{L-1}) , by using the cumulative density function as a transform function. Let's define a transform function f(x) based on the cumulative density function as

$$f(x) = X_0 + (X_{L-1} - X_0)\, C(x) \quad (3)$$

Then the output image of the HE, Y= {Y(i, j)}, can be expressed as

$$Y = f(X) \quad (4)$$

$$= \{f(X(i,j) \mid \forall X(i,j) \in X\} \quad (5)$$

The high performance of the HE in enhancing the contrast of an image as a consequence of the dynamic range expansion, Besides, HE also flattens a histogram. HE can introduce a significant change in brightness of an image, which hesitates the direct application of HE scheme in consumer electronics. For instance, Fig. 1(a) and fig. 1(b) shows as original gray scale image with its histogram and the resultant image of the HE that are composed of 256 gray levels. Fig. 2(a) and fig. 2(b) shows as original color image with its histogram and the resultant image of the HE that are composed of RGB components. Observe that here the equalized image is much darker than the input image and unnatural enhancement in most part of the image. Note that the HE maps its input gray to a output gray level.

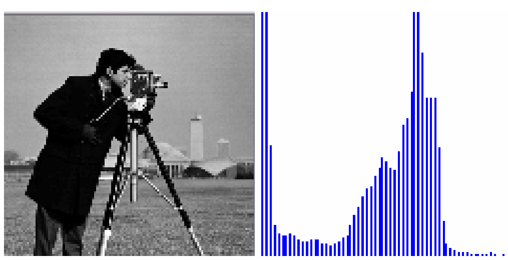

Fig 1(a) Original Gray Scale Image and its Histogram

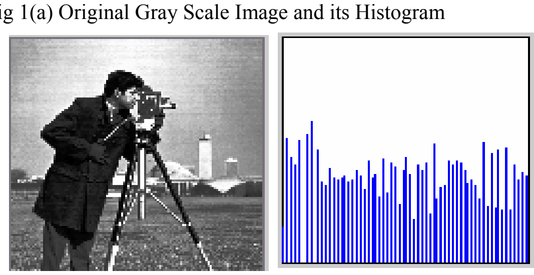

Fig 1(b) Result of HE & its histogram

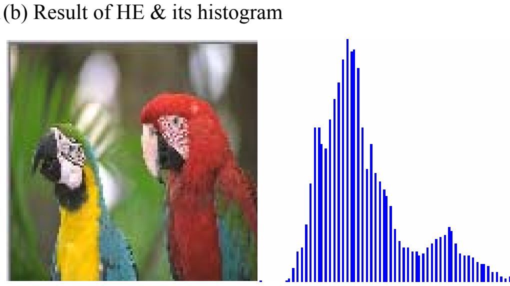

Fig 2(a) Original color Image and its Histogram

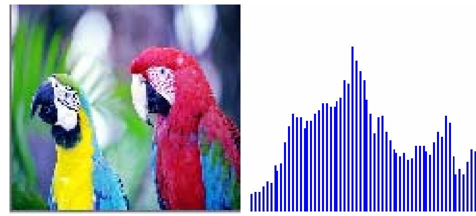

Fig 2(b) Result of HE & its histogram

B. Brightness Preserving Bi-Histogram Equalization

Let **Xm** be the mean of the image X and assume that $X_m \varepsilon \{X_0, X_1 \ldots . X_{L-1}\}$. Based on the mean, the input image is decomposed into two sub-images X_L and X_U as

$$X = X_L \cup X_U \quad (6)$$

Where

$$X_L = \{(X(i,j) \mid X(i,j) \le X_m, \forall X(i,j) \in X\} \quad (7)$$

and

$$X_U = \{(X(i,j) \mid X(i,j) > X_m, \forall X(i,j) \in X\} \quad (8)$$

Note that the sub-image X_L is composed of $\{X_0, X_1 \ldots . X_m\}$ and the other image X_U is composed of $\{X_{m+1}, X_{m+2}, \ldots .. X_{L-1}\}$. Next, define the respective probability density functions of the sub-images X_L and X_U as

$$P_L(X_k) = n^k_L / n_L \quad (9)$$

Where k = 0,1…., m, and

$$P_U(X_k) = n^k_u / n_u \quad (10)$$

Where k = m+1, m+2, …… ,L-1, in which n^k_L and n^k_U represent the respective numbers of X_k in $\mathbf{X}_L$ and X_U, and n_L and n_u are the total number of samples in X_L and X_U, respectively.

Note that $n_L = \sum_{k=0}^{m} n^k{}_L$, $n_u = \sum_{k=m+1}^{L-1} n^k{}_u$ n^k_u and $n=n_L + n_u$. The respective cumulative density functions for X_L and X_u are then defined as

$$C_L(x) = \sum_{j=0}^{k} P_L(X_j) \quad (11)$$

and

$$C_u(x) = \sum_{j=m+1}^{k} P_u(X_j) \quad (12)$$

where $X_k = X$ Note that $C_L(X_m) = 1$ and $c_u(X_{L-1}) = 1$ by definition.

Similar to the case of HE where a cumulative density function is used as a transform functions, let's define the following transform functions exploiting the cumulative density functions

$$f_L(x) = X_0 + (X_m - X_0)\, C_L(x) \quad (13)$$

and

$$f_u(x) = X_{m+1} + (X_{L+1} - X_{m+1})\, C_U(x) \quad (14)$$

Based on these transform functions, the decomposed sub-images are equalized independently and the composition of the resulting equalized sub-images constitute the output of BBHE.That is, the output image of BBHE, Y, is finally expressed as

$$Y = \{Y(i,j)\} \quad (15)$$

$= f_L(X_L) \cup f_u(X_u),$ (16)

Where

$$f_L(X_L) = \{f_L(X(i,j) \mid \forall X(i,j) \in X_L\} \quad (17)$$

and

$$f_u(X_u) = \{f_u(X(i,j) \mid \forall X(i,j) \in X_u\} \quad (18)$$

Note that $0<C_L(x)$, $C_U(x)< 1$,it is easy to see that $f_L(X_L)$ equalizes the sub-image X_L over the range (X_o, X_m) whereas $f_U(X_U)$ equalizes the sub-image X_U over the range (X_{m+1},X_{L-1}). As a consequence ,the input image X is equalized over the entire dynamic range (X_o,X_{L-1}) with the constraint that the sample less than the input mean are mapped to $(X_o, X_m)$and the samples greater than the mean are mapped to (X_{m+1},X_{L-1}).

C. Analysis on The Brightness Change By the BHE

Suppose that X is a continuous random variable, then the output of the HE, Y is also regarded as a random variable. It is well known that the HE produces an image, whose gray levels have a uniform density, i.e.,

$P(x) = 1 |(X_{L-1} +X_o)$ (19)

for $X_o< x < X_{L-1}$.Thus, it is easy to show that the mean brightness of the output image of the HE is the middle grey level since

$$E(Y) = \sum xP(x)dx \quad (20)$$

$$= \sum_{x_0}^{x_{L-1}} \frac{x}{X_L - X_0} dx \quad (21)$$

$$= \frac{X_L + X_0}{2} \quad (22)$$

Where E(Y) denotes statistical expectation . It should be emphasized here that the output mean of the HE has nothing to do with the input message That is , it is always the middle gray level to no matter how much the input image is bright/dark. clearly, this property is not desirable in many applications.

Suppose that X is a random variable, which has symmetric distribution around its mean X_m. When sub-images are equalized independently, the mean brightness of the output of the BHE can be expressed as

$E(Y) = E(Y| X \le X_m)Pr(X \le X_m)+E(Y |X >X_m)Pr(X>X_m)$

$= 1/2\{E(Y |X \le X_m)+E(Y| X >X_m)\}$ (23)

where $Pr(X \le X_m) = Pr(X > X_m)= 1/2$ .Since X is assumed to have a symmetric distribution around X_m. With similar discussion used to obtain (22), it can easily shown that

$E(Y|X \le X_m)=(X_0+X_m)/2$ (24)

and

$E(Y| X > X_m)=(X_{m+1}+X_{L-1})/2$ (25)

Substituting (24) and (25) in (23) results in

$E(Y)= (X_m + X_g) / 2$ (26)

Where

$X_g=(X_0 + X_{L-1}) / 2$ (27)

X_g is the middle gray level, which implies that the mean brightness of the equalized image by BHE locates in the middle of the input mean the middle gray level. Note that the output mean of the BHE is a function of the input mean brightness X_m. This fact clearly indicates that the BHE preserves the brightness compared to HE where output mean is always the middle gray level. In some images, this level of brightness preservation is not sufficient to avoid unpleasant artifacts. Fig 3(a) and 3(b) shows the result of BHE on gray scale and color image . They clearly show that higher degree of brightness preservation is required for these images to avoid unpleasant artifacts.

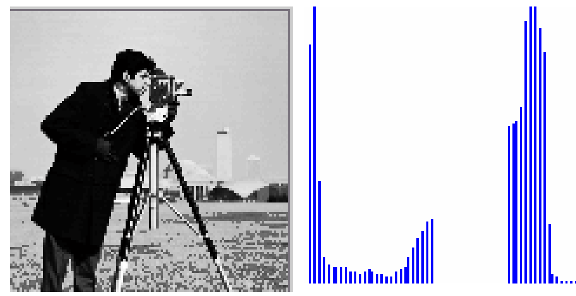

Fig. 3(a) Result of BHE of gray scale image and its histogram

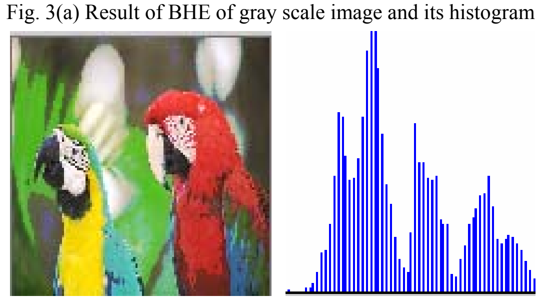

Fig. 3(b) Result of BHE & its histogram

D. RECURSIVE MEAN – SEPARATE HISTOGRAM EQUALIZATION

Brightness preservation can be achieved by having mean – separation before performing the equalization process as in BHE. Mean-separation refers to separating the image into two sub-image based on the mean of the input image. This is equivalent to separate the histogram into two based on the mean of the input images histogram. After mean separation, the resulting new histograms are equalized independently. In typical HE, no mean-separation is performed and thus, no brightness preservation, In BHE, the mean-separation is done once and thus, achieve certain extends of brightness preservation. If more brightness preservation is required, a new technique is proposed to perform the mean separation recursively, separate the resulting histograms again based on their respective means. Recursive Mean – Separate Histogram Equalization (RMSHE) proposed in this paper for color images is basically a generalization of HE and BHE in the aspect of brightness preservation [7]. This idea is illustrated more clearly in the following figures (4 & 5). It will be shown that more mean-separation recursively will result in more brightness preservation.

Fig. 4 shows histogram before and after HE. As been shown in (22), the output means E (Y) of typical HE is given as follows:

$E(Y) = (X_0+X_{L-1}) / 2 = X_g$ (28)

No mean – separation is performed before equalization and hence, no brightness preservation. This is indicated in (28). (i.e.) HE is equivalent to RMSHE with recursion level, r = 0.

Fig.5 shows histogram before and after BHE. As been shown in (26), the output mean E (Y) after BHE is as follows:

$E (Y) = (X_m + X_g) / 2$ (29)

One mean-separation is performed before equalization and hence, some level of brightness preservation. This is indicated by equation (29). (i.e.) BHE is equivalent to RMSHE with recursion level r=1. In order to achieve higher brightness preservation, this paper proposed to perform the mean separation recursively; separate the resulting histograms again based on their respective means.

Supposed that X is further separated into 4 portions based on the mean of the two new histograms, X_{ml} and X_{mu} as shown in Fig. 5.

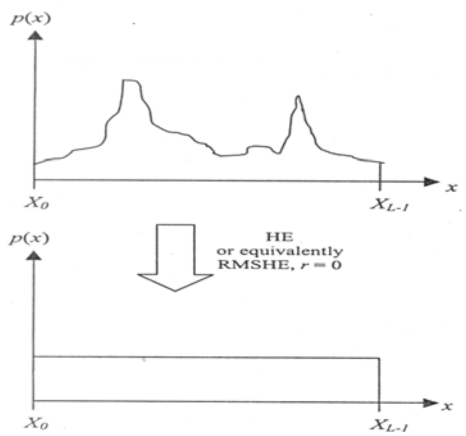

Fig.4. Histogram before and after HE or equivalently, RMSHE,r=0

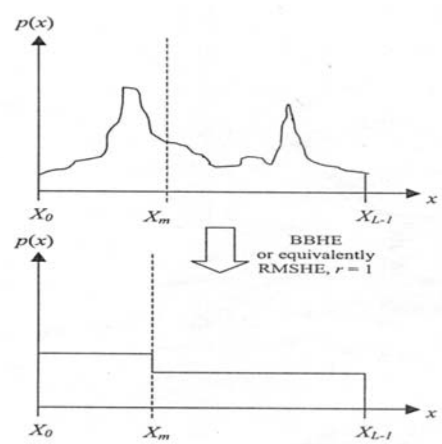

Fig.5. Histogram before and after BHE or equivalently, RMSHE, r=1

$$X_{ml} = \frac{\int_{x_0}^{x_m} xP(x)dx}{\int_{x_0}^{x_m} P(x)dx} = 2\int_{x_0}^{x_m} xP(x)dx \quad (29)$$

$$X_{mu} = \frac{\int_{x_{m+1}}^{x_{L-1}} xP(x)dx}{\int_{x_{m+1}}^{x_{L-1}} P(x)dx} = 2\int_{x_{m+1}}^{x_{L-1}} xP(x)dx \quad (30)$$

Where

$$\int_{x_0}^{x_m} P(x)dx = \int_{x_{m+1}}^{x_{L-1}} P(x)dx = \frac{1}{2} \quad (31)$$

Because X is assumed to have a symmetric distribution around X_m .

Fig. 6 shows the histogram before and after equalizing the four portions of the histogram independently. This is the result of RMHSE with recursion level, r=2. The following shows the formulation of the output mean.

$E (Y)= E (Y \mid X_{m1})Pr(X \le X_m)$
$+ E (Y \mid X_{ml} < X \le X_m) Pr (X_{ml} < X \le X_m)$
$+ E (Y \mid X_m < X \le X_{mu}) Pr (X_m < X \le X_{mu})$
$+ E (Y \mid X > X_m) Pr (X > X_{mu})$
$=1/4 \{ + E (Y \mid X_{ml} < X \le X_{ml} X_{ml} < X \le X_m)$
$+ E (Y \mid X_m < X \le X_{mu}) +E (Y> X_{mu})\}$

(32)

where $Pr(X\le X_{ml}) = Pr(X_{ml}< X \le X_m) = Pr(X_m<X \le X_{mu}) = Pr (X >X_{mu}) = ¼$ is used since X is assumed to have a symmetric distribution around X_m with similar discussion to obtain (22)

$E(Y) = ¼ \{ [(X_0 + X_{ml}) /2] + (X_{ml} + X_m)/2]$
$+ [(X_m + X_{mu})/2 + [(X_{mu} + X_{L-1}) / 2] \}$
$= ¼ \{ [(X_0 + X_{L-1}) /2] + (2(X_{ml} + X_{mu}) / 2] + X_m\}$
$= ¼ \{ X_G + 2X_m + X_m\}$
$= ¼ \{ X_G + 3X_m \}$

(33)

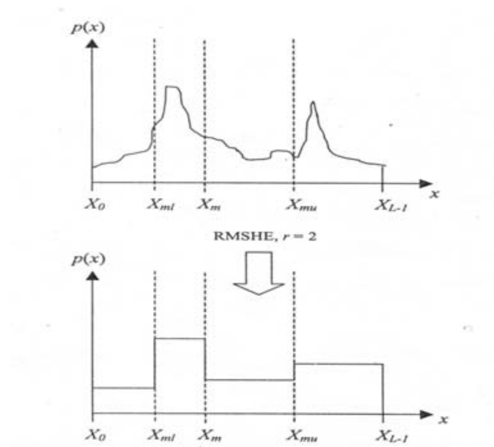

Fig. 6. Histogram before and after RMHSE, r=2

From (29) and (30)

$$X_{mu} + X_{ml} \;/\; 2 = 2\int_{x_0}^{x_m} xP(x)dx + 2\int_{x_{m+1}}^{x_{L-1}} xP(x)dx \;/\; 2$$

$$= \int_{x_0}^{x_m} xP(x)dx + \int_{x_{m+1}}^{x_{L-1}} xP(x)dx \qquad (34)$$

$$= \int_{x_0}^{x_{L-1}} xP(x)dx$$

$$= X_m$$

Two mean-separations before equalizations results in higher level of brightness preservation compare to zero and one mean-separations as in the case of HE and BHE respectively. This is indicated by equation (33) where the weight of input mean, X_m has increased to three times as much as the weight of middle gray level, X_g. Following the above discussion, it is very reasonable to expect that the brightness preservation will increase as the number of recursive mean-separations increases. Based on the observations on the output mean for RMHSE recursion level r=0,1 and 2 shown above, it is not difficult to generalized the output mean E (Y) for RMSHE recursion level r=n as follows

$r = 0$, $E(Y) = X_G$

$r = 1$, $E(Y) = (X_m + X_G) / 2$

$r = 2$, $E(Y) = (3X_m + X_G) / 4$ ….

$r = n$, $E(Y) = ((2^n - 1) + X_m + X_G) / 2^n$

$= X_m - [(X_G - X_m) / 2^n]$ (35)

Equation (35) indicates that as the recursion level, n grows larger; E (Y) will eventually converge to the input mean, X_m. In applications of consumer electronics, the variety of image involve are too wide to be covered with only a specific level of brightness preservation. Therefore, the scalability in this algorithm is the most desirable property to allow adjustment of level of brightness preservation base on individual image's requirement.

III. Performance Analysis

Image fidelity criteria are useful for measuring image quality and for rating the performance of a processing system. There are two types of criteria that are used for evaluation of image quality: subjective and quantitative. The subjective criteria use rating scales such as goodness scales and impairment scales. The overall goodness criterion rates image quality on a scale ranging from excellent to unsatisfactory. The impairment scale rates an image on the basis of the level of degradation present in an image when compared with an ideal image.

Among the quantitative measures, a class of criteria used is called the mean square criterion. It refers to some sort of average or sum of the squares of the error between two images. In many applications, the mean square error is expressed in terms of Signal to Noise Ratio (SNR) which is defined in decibels as

SNR = 10 log $_{10}\sigma^2 / \sigma_e^2$

Where σ^2 is the variance of the desired image.

Another definition of ASNR (Average Signal to Noise Ratio), used commonly in image enhancement applications, is

ASNR = (f – b)/σ

Where **f** is the average gray-level value of the enhanced image

b is the mean gray-level value of the original image.

σ is the standard deviation

If the ASNR value is larger, the enhancement method performs better.

Table 6.1 : Performance of various histogram equalization techniques on gray scale image

Methods	***ASNR value***
Histogram Equalization	*0.65*
Bi-histogram Equalization	*0.77*
Recursive Mean-Separate Histogram Equalization	*0.93*

Table 6.2 : Performance of various histogram equalization techniques on color image

Methods	***ASNR value***
Histogram Equalization	*0.61*
Bi-histogram Equalization	*0.74*
Recursive Mean-Separate Histogram Equalization	*0.83*

IV. Results

In order to demonstrate the performance of the proposal algorithm, simulation results of RMSHE on gray scale and color image is presented in Fig. 7(a) and 7(b). In the case of image, the image resulting from HE, and BHE (review fig. 2(a), 2(b),3(a) and 3(b)) have mean brightness much different from the original image and hence, result in unnatural contrast enhancement. Result from RMSHE with r=2 (fig. 7(a) and 7(b)) clearly show that the RMSHE algorithm has increased the brightness preservation and yielded a more natural enhancement.

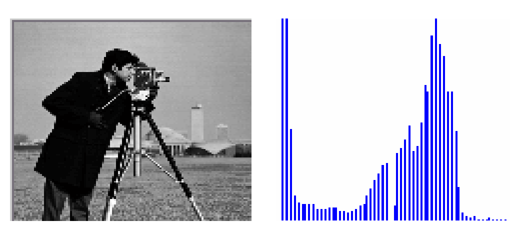

Fig. 7(a) Result of RMSHE r=2 of gray scale image and its histogram


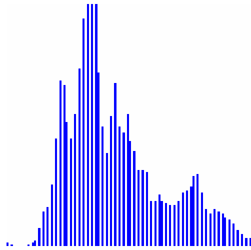

Fig. 7(b)Result of RMSHE r=2 of color image and its histogram

V. CONCLUSION

In this paper, a new contrast enhancement algorithm referred as the recursive mean-separate histogram equalization (RMSHE) with scalable brightness preservation is proposed and performance of this algorithm can be measured using ASNR ratio. The RMSHE is a generalization of HE and BHE in term of brightness preservation. The main idea lies on recursively separating the input histogram based on the mean. The ultimate goal behind in the RMSHE is to allow higher level of brightness preservation to avoid unwanted noises.

The analysis shows that the output mean will converge into the input mean as the number of recursive mean-separation increases. Therefore, the scalability in this algorithm is the most desirable property especially in consumer electronics to allow scaling of brightness preservation, suited to individual image. A comparison study is made on various histogram equalization techniques and performance of all has been measured.

VI. REFERENCES

[1] Scott E. Umbauugh , Computer Vision and Image Processing, PH , New Jersey 1998, pp209.

[2] Yeong-Taeg Kim , "Contrast Enhancement using Brightness Preserving Bi- Histogram equalization", IEEE trans. on consumer Electronics, Vol. 43 , 1998.

[3] Yu wan , Qian chen and Bao- Min Zhang , " Image enhancement based on equal area Dualistic sub image histogram equalization method", IEEE Trans. on consumer Electronics, Vol. 45, 1999.

[4] Young – Tack Kim and Yong-Hun Cho , "Image enhancing method using Mean separate Histogram equalization", U.S. Patent, Patent No.5, 963,665, 1999.

[5] Yeong-Taeg Kim "Method for image enhancing using Quantized Mean separate Histogram Equalization", U.S. Patent , Patent No.5,857,033, 1999

[6] Yeong-Taeg Kim "Image Enhancing method and circuit using Mean separte / quantized Mean separate Histogram equalization and color compensation", U.S. Patent, Patent No.6,049,626, 2000.

[7] Soong-Der Chen, Ramli,A.R, "Contrast enhancement using recursive mean-separate histogram equalization for scalable brightness preservation" Consumer Electronics, IEEE Transactions on Volume 49, Issue 4, Nov. 2003 Page(s): 1301 - 1309

VII. BIOGRAPHIES

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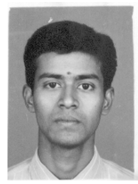

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